On the structure of the set of active sets in constrained linear-quadratic optimal control

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Abstract

Optimal control problems are complex. This is the case, among other reasons, because they are infinite-dimensional problems. On the other hand, solutions to optimal control problems are often surprisingly simple and they can be described with finitely many parameters. This is evident from problems with bang-bang solutions, for example.

The unconstrained linear-quadratic regulator is another classical example for an infinite-dimensional problem with a particularly simple solution (a finite-dimensional state feedback law $u = Kx$). In the constrained case, the solution turns into a piecewise affine law. While structurally still simple, this piecewise affine law is already so complex that it is usually not useful as a closed-form solution (in contrast to $u = Kx$). In fact, the number of pieces is a function of the horizon N (in contrast to $u = Kx$ that is independent of N), the number of pieces grows often dramatically with N, the solution for a horizon N is not in general contained in that for $N + 1$, and it is not clear if the limit $N \to \infty$ exists.

We first recall the solution of the constrained case can be characterized by the set of active sets of the underlying quadratic program. We then show there exists a simple structure in the set of active sets that can be used to analyze the solution as a function of the horizon. Specifically, we show every active set for horizon $N + 1$ contains an active set for horizon N. Moreover, it is easy to detect which affine pieces of the solution for horizon N actually do persist for $N + 1$, $N + 2$, … and which do not, which simplifies the analysis of the limit $N \to \infty$.

Apart from the insights into the solution structure, the proposed analysis method provides opportunities for improving existing methods for the computation of explicit model predictive control solutions. We will show there also exist opportunities for improving online model predictive control methods if the set of all active sets is not calculated a priori.

Since the analysis is not based on geometric objects such as polytopes and affine laws, there may be potential for an extension of the proposed approach to nonlinear cases, which are briefly mentioned in an outlook. Moreover, it is worthwhile to investigate the robustness properties of the solution (and thus model predictive control) based on the active set structure.

Short Biography

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